Generation and fidelity of entangled states with trapped ions in a lossy cavity

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Abstract

We analyse the role of dissipation in a system consisting of a single trapped ion in a lossy cavity. In this work, we apply the small-loss rate expansions for the density operator [1] in order to show how the generation of entangled states are affected by the cavity losses. In that system, we estimate the efficiency of our method of generation of Bell states [2], including some considerations about imperfections (thermal noise) in the initial preparation of the system.

The study of quantum correlations and the ability to coherently control entanglement in a specific physical system have recieved increasing attention in the last years [3]. They have a wide range of applications both in quantum information and in topics of fundamental interest in quantum optics such as the comprehension of the decoherence phenomenon [4].

We consider a single two level ion trapped in a harmonic potential also in interaction with a single-mode electromagnetic field in a lossy cavity [5]. We assume that the ion is cooled to the Lamb-Dicke regime where the spatial extension of its wave packet is smaller than the wavelenght of the cavity mode. In particular, we consider that the frequency of the field is exactly equal to the frequency gap between the two internal (electronic) levels of the ion minus its harmonic motional frequency, what is often called the blue sideband regime [6]. The density operator for that system obeys the following master equation in the interaction picture

$$\frac{\partial\hat{\rho}}{\partial t} = -\frac{i}{\hbar}\left[\hat{H},\hat{\rho}\right] + \hat{\mathcal{L}}\hat{\rho} \tag{1}$$

where

$$\hat{H} = \hbar\,\eta\,g\left(\hat{\sigma}_{+}\hat{a}\,\hat{b} + \hat{\sigma}_{-}\hat{a}^{\dagger}\hat{b}^{\dagger}\right) \tag{2}$$

and

$$\hat{\mathcal{L}}\hat{\rho} = \frac{\kappa}{2}\left(2\,\hat{b}\,\hat{\rho}\,\hat{b}^{\dagger} - \hat{b}^{\dagger}\hat{b}\,\hat{\rho} - \hat{\rho}\,\hat{b}^{\dagger}\hat{b}\right), \tag{3}$$

being $\hat{a}\,(\hat{a}^{\dagger})$ the creation (annihilation) operator of phonons in the harmonic motion of the trapped ion, $\hat{b}\,(\hat{b}^{\dagger})$ the creation (annihilation) operator of photons in the quantised field, $\hat{\sigma}_{+}\,(\hat{\sigma}_{-})$ the internal electronic raising (lowering) operator of the two level ion, g the coupling constant between the cavity field and the trapped ion, η the Lamb-Dicke parameter, and κ is the loss-rate decay constant of photons (linewidth of the cavity mode).

There are many distinct approaches to solve the master equation (1). One can, for instance, make use of the phase space approach and solve the resulting Focker-Plank equation [7]. We are going to use a distinct approach here. Our goal is to analyse the first correction with regard to κ in the unitary evolution of the system when dissipation is taken into account. In that case, a suitable way to achieve this is considering a density operator expansion in powers of that parameter [1]. So, we can write

$$\hat{\rho} = \hat{\rho}_0 + \kappa\,\frac{\partial\hat{\rho}}{\partial\kappa} + \frac{\kappa^2}{2!}\,\frac{\partial^2\hat{\rho}}{\partial\kappa^2} + ..., \tag{4}$$

where the sucessive coeficients may in general be calculated by substituing (4) into the master equation (1). Following that procedure we find, for $n = 0$

$$\frac{\partial\hat{\rho}_0}{\partial t} = -\frac{i}{\hbar}\left[\hat{H},\hat{\rho}_0\right] \tag{5}$$

and for $n \geq 1$

$$\begin{aligned}\frac{\partial^n \hat{\rho}}{\partial \kappa^n} &= -\frac{i}{\hbar}\left[\hat{H}, \frac{\partial^n \hat{\rho}}{\partial \kappa^n}\right] \\ &+\frac{n}{2}\left(2\hat{b}\,\frac{\partial^{n-1}\hat{\rho}}{\partial \kappa^{n-1}}\,\hat{b}^\dagger - \hat{b}^\dagger\hat{b}\,\frac{\partial^{n-1}\hat{\rho}}{\partial \kappa^{n-1}} - \frac{\partial^{n-1}\hat{\rho}}{\partial \kappa^{n-1}}\,\hat{b}^\dagger\hat{b}\right)\end{aligned} \tag{6}$$

In order to solve the problem, we have to obtain the unitary evolution (5). After that we are able to calculate the corrections (6). Depending on the values of κ, the accurate solution might demand the inclusion of more terms, or in a extreme case, the entire series is needed. Some simple considerations about the convergence of the method is presented in [1].

The use of expansion (4) allows one to calculate any expectation value as well as the state fidelity in case of a state generation method. We recall that a possible distance measure between a mixture state $\hat{\rho}$ and a pure state $|\psi\rangle$ is given by the fidelity $F = \sqrt{\langle\psi|\hat{\rho}|\psi\rangle}$.

It is well known [2] that for suitable initial preparations of the system it is possible to generate the entire Bell basis involving fock states of both the cavity field and the vibrational motion. In particular, when the cavity and the vibrational motion are both in the vacuum state and for a ideal lossless cavity it is possible to generate the following Bell state

$$|\psi\rangle = \frac{|0\rangle_f|0\rangle_v + |1\rangle_f|1\rangle_v}{\sqrt{2}}. \tag{7}$$

The generation of the Bell state (7) occurs in definite times given by $t_k = \pi(4k+1)/2\eta g$ $(k = 0, 1, 2, ...)$. When $\kappa = 0$, the ideal lossless case, the fidelity is equal to the unity. By the other hand, when dissipation is taken into account, this fidelity tends to decrease due to the coupling with the environment. Furthermore, the unitary fidelity is just reached in case of perfect preparation of the quantised field as well as the centre-of-mass motion in the vacuum state. In this contribution, we analise this matters in the framework of the expansion (4), keeping only linear terms in κ what seems to be a suitable approximation in case the dissipation is much smaller than the ion-field coupling.

Firstly, the effect of dissipation in the cavity will be presented. One of the features to be shown is that for perfect initial states (vacuum for both cavity and motional state) and for small values of the parameter $\Gamma = \kappa/\eta g$, the decrease of the fidelity due to dissipation is negligible.

Secondly, we consider some initial thermal noise instead of perfect vacuum. It is made by means of the thermal state given by

$$\hat{\rho} = \sum_{n=0}^{\infty} \frac{\bar{n}^n}{(\bar{n}+1)^{n+1}}|n\rangle\langle n|, \tag{8}$$

where $\bar{n}$ is the mean number of either photons in the cavity or phonons in the motion of the trapped ion.

As it is going to be presented, the action of dissipation on the fidelity will be different whether the noise is either in the cavity or in the motion. Actually, the effect of this imperfections in the initial preparation of the system is more important than the first order contribution of the cavity losses themselves. In fact, the presence of thermal excitations in the cavity mode or in the vibrational mode of the ion causes a quick decrease in the fidelity.

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