Families of higher-generation Schrödinger cat states in a cavity: generation and properties

M.V. Dantas[*,†], *J.M.C. Malbouisson*[*], *and B. Baseia*[‡]

[*]**Instituto de Física, Universidade Federal da Bahia, 40210-340, Salvador, BA, Brazil**
[†]**Departamento de Física, Universidade Estadual de Feira de Santana, BA, Brazil**
[‡]**Instituto de Física, Universidade Federal de Goiás, 74001-970, Goiânia, GO, Brazil**

Abstract

We discuss the production of families of higher-generation Schrödinger-cat states in a high-Q cavity and study their nonclassical properties.

Production of families of higher-generation Schrödinger-cat states in a cavity can be conceived as a slight extension of the procedure to generate simple Schrödinger cats [1], like the even and the odd coherent state. This experimental scheme, illustrated in Fig. 1, is based on the quantum nondemolition procedure, using Ramsey atomic interferometry and dispersive atom-field interaction in a high-Q superconducting microwave cavity, allowing for the manipulation of photon states in it. As illustrated, the first part of the system consists of a source of two-level Rydberg atoms, with circular states $|g\rangle$ and $|e\rangle$ (corresponding to the levels 50 and 51 in the case of rubidium), provided with a velocity selector and an exciting laser beam promoting the atoms to their excited state $|e\rangle$. Then, before crossing the high-Q cavity, the atoms pass by a low-Q cavity (the first Ramsey zone R1) containing a classical field where they are prepared at equally weighted superpositions of the states $|g\rangle$ and $|e\rangle$. At the high-Q cavity (C), possessing initially a coherent state with chosen frequency and intensity, the dispersive atom-field interaction entangles the atomic and field state components which is rearranged after the atom crosses a second Ramsey zone (R2) also fixed to produce a $\pi/2$ rotation in the atomic Hilbert space. Now, by detecting the atom in either the ground or the excited state, the field state in the cavity is projected into superpositions of the initial coherent state with a phase shifted partner bearing the same coefficients as those of the atomic superposition before the atom enters the cavity added eventually by a relative phase.

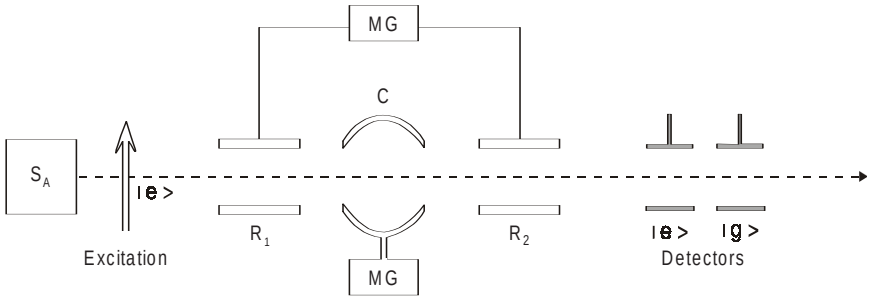


Figure 1: Experimental setup to produce Schrödinger-cat states in a high-Q cavity.

In fact, consider that the cavity frequency is adjusted close to resonance with the transition $|e\rangle \rightarrow |i\rangle$ ($n = 52$), detuned by Δ, but very distinct of that associated with the $|g\rangle \rightarrow |e\rangle$ transition. This dispersive interaction between the atom and the cavity field can be described by the effective Hamiltonian [2]

$$\hat{H}_{int} = \hbar\omega_{eff}\,\hat{a}^{\dagger}\hat{a}\,\left(|i\rangle\langle i| - |e\rangle\langle e|\right)\ , \tag{1}$$

where $\omega_{eff} = 2\mathbf{d}^2/\Delta$ and $\mathbf{d}$ is the atomic dipole moment. This implies that, as the atom crosses the cavity in a superposition of the states $|g\rangle$ and $|e\rangle$, the dispersive interaction produces a phase shift ϕ in the cavity field when the atomic state is $|e\rangle$ but has no effect when the atom is in the state $|g\rangle$, that is

$$(c_g\ |g\rangle + c_e\ |e\rangle)\ |\Psi_F\rangle \rightarrow c_g\ |g\rangle\ |\Psi_F\rangle + c_e\ |e\rangle\ \left(\exp[i\phi\,\hat{a}^\dagger\hat{a}]\ |\Psi_F\rangle\right)\ , \tag{2}$$

where $\phi = \omega_{eff}\,t_c$, t_c being the time the atom takes to cross the cavity. Now, assume that the field in the cavity is initially in the coherent state $|\alpha\rangle$ (α real, without loss of generality) and that a first atom enters C in the superposition $(|g\rangle + |e\rangle)/\sqrt{2}$; then, since $\exp[i\phi_1\hat{a}^\dagger\hat{a}]\,|\alpha\rangle = \left|e^{i\phi_1}\alpha\right\rangle$, the entangled atom-field state, after R2 where $|e\rangle \rightarrow (|g\rangle + |e\rangle)/\sqrt{2}$ and $|g\rangle \rightarrow (|g\rangle - |e\rangle)/\sqrt{2}$, becomes $|g\rangle\ \left(\left|e^{i\phi_1}\alpha\right\rangle + |\alpha\rangle\right) + |e\rangle\ \left(\left|e^{i\phi_1}\alpha\right\rangle - |\alpha\rangle\right)$. If the atomic velocity is previously selected such that $\phi_1 = \pi$, detection of the atom projects the cavity field into the states $\mathcal{N}_1^{\pm}\left(|-\alpha\rangle \pm |\alpha\rangle\right)$, with + and − referring to detection of $|g\rangle$ and $|e\rangle$ respectively. These even and odd coherent states, if $|\alpha|$ is large, correspond to the first-generation Schrödinger cats.

To obtain the following generation one needs to pass a second atom through the system, with velocity fixed such that $\phi_2 = \pi/2$, assuming that the state inside the cavity corresponds to one of the first generation states obtained upon the detection of the first atom. In this case, detecting the second atom projects the field in the cavity in one of the states $[(|-\alpha\rangle + \epsilon_1\,|\alpha\rangle) \pm (|-i\alpha\rangle + \epsilon_1\,|i\alpha\rangle)]$, with + and − referring to detection of $|g\rangle$ and $|e\rangle$ respectively, and with $\epsilon_1 = \pm 1$, depending on the detection of the first atom. The second generation thus obtained are the superposition of the even or odd coherent states with a partner rotated by $\pi/2$ in the α-plane. One can go on by passing a third atom with velocity selected such that $\phi_3 = \pi/4$, the field in the cavity being in aone of the second-generation states, to get third-generation states; these will be superpositions of a second-generation state with another rotated by $\pi/4$ in the α-plane.

The scheme described above can be continued and one obtains, after sequential passage of N atoms with well defined velocities such that $\phi_k = \pi/2^{k-1}$ for $k = 1, 2, ..., N$ and detecting all of them, the state

$$|\Psi_{\epsilon_1\ldots\epsilon_N}(\alpha)\rangle = \frac{\mathcal{N}_{\epsilon_1\ldots\epsilon_N}(\alpha)}{\mathcal{N}_{\epsilon_1\ldots\epsilon_{N-1}}(\alpha)}\left(\left|\Psi_{\epsilon_1\ldots\epsilon_{N-1}}\left(e^{i\pi/2^{N-1}}\alpha\right)\right\rangle + \epsilon_N\left|\Psi_{\epsilon_1\ldots\epsilon_{N-1}}(\alpha)\right\rangle\right)\ , \tag{3}$$

where the normalization constant is given by

$$\mathcal{N}_{\epsilon_1\ldots\epsilon_N}(\alpha) = \left[2^{2N}\exp(-\alpha^2)\mathcal{F}_{\epsilon_1\ldots}\left(\alpha^2\right)\right]^{-\frac{1}{2}}\ , \tag{4}$$

with the functions $\mathcal{F}$ satisfying the recurrence relations

$$\mathcal{F}_{\epsilon_1\ldots\epsilon_N}\left(\alpha^2\right) = \frac{1}{2}\left[\mathcal{F}_{\epsilon_1\ldots\epsilon_{N-1}}\left(\alpha^2\right) + \frac{1}{2}\epsilon_N \mathrm{Re}\left\{\mathcal{F}_{\epsilon_1\ldots\epsilon_{N-1}}\left(e^{i\pi/2^{N-1}}\alpha^2\right) + \mathcal{F}_{\epsilon_1\ldots\epsilon_{N-1}}\left(-e^{i\pi/2^{N-1}}\alpha^2\right)\right\}\right] \tag{5}$$

with $\mathcal{F}_{\epsilon_1}\left(\alpha^2\right) = \frac{1}{2}\left[\exp(\alpha^2) + \epsilon_1\exp(-\alpha^2)\right]$. Here, $\epsilon_k = \pm 1$ depending on whether the k-th atom is detected in $|g\rangle$ or in $|e\rangle$ states respectively. One sees then the N-generation possesses 2^N distinct members, each one corresponding to a definite sequence of atomic detections. Two of these families of states, those corresponding $\epsilon_k = 1$ for all $k = 2, 3, ..., N$, has been discussed before [3]; in the present work, some new interesting aspects and properties, which appear in this more general situation, are explored.

The photon-number distribution of such states are given by

$$P_{\epsilon_1\ldots\epsilon_N}(n;\alpha) = \frac{1}{\mathcal{F}_{\epsilon_1\ldots\epsilon_N}(\alpha^2)}\frac{\left(\alpha^2\right)^n}{n!}A_{\epsilon_1\ldots\epsilon_N}(n)\ , \tag{6}$$

where

$$A_{\varepsilon_1\ldots\varepsilon_N}(n) = \left|\frac{1}{2^N}\prod_{j=1}^{N}\left[e^{in\pi/2^{j-1}}) + \epsilon_j\right]\right|^2\ . \tag{7}$$

It is interesting to notice that all the states in these families have well defined photon-number parities, characterized by the detection of the first atom; that is, all the states for which $\epsilon_1 = +1$ have only number

states with even number of photons occupied while those with $\epsilon_1 = -1$ possess solely odd number states occupied.

From Eq. (6) one easily obtain the Taylor expansions of the normalization functions $\mathcal{F}$, namely

$$\mathcal{F}_{\epsilon_1 \ldots \epsilon_N}\left(\alpha^2\right) = \sum_{n=0}^{\infty} \frac{\left(\alpha^2\right)^n}{n!} \mathcal{A}_{\varepsilon_1 \ldots \varepsilon_N}(n) \ , \tag{8}$$

from which one shows that

$$\langle \widehat{n} \rangle_{\epsilon_1 \ldots \epsilon_N} = \sum_{n=0}^{\infty} n P_{\epsilon_1 \ldots \epsilon_N}(n;\alpha) = \alpha^2 \, \frac{\mathcal{F}^{(1)}_{\epsilon_1 \ldots \epsilon_N}\left(\alpha^2\right)}{\mathcal{F}_{\epsilon_1 \ldots \epsilon_N}\left(\alpha^2\right)} \ , \tag{9}$$

$$\left\langle \widehat{n}^2 \right\rangle_{\epsilon_1 \ldots \epsilon_N} = \sum_{n=0}^{\infty} n^2 P_{\epsilon_1 \ldots \epsilon_N}(n;\alpha) = \langle \widehat{n} \rangle_{\epsilon_1 \ldots \epsilon_N} + \alpha^4 \, \frac{\mathcal{F}^{(2)}_{\epsilon_1 \ldots \epsilon_N}\left(\alpha^2\right)}{\mathcal{F}_{\epsilon_1 \ldots \epsilon_N}\left(\alpha^2\right)} \ , \tag{10}$$

where $\mathcal{F}^{(m)}_{\epsilon_1 \ldots \epsilon_N}\left(\alpha^2\right)$ denotes the m-th derivative of $\mathcal{F}_{\epsilon_1 \ldots \epsilon_N}\left(\alpha^2\right)$ with respect to α^2. As a consequence, the Mandel factor for these states is given by

$$Q_{\epsilon_1 \ldots \epsilon_N}\left(\alpha^2\right) = \frac{\left\langle \widehat{n}^2 \right\rangle_{\epsilon_1 \ldots \epsilon_N}}{\langle \widehat{n} \rangle_{\epsilon_1 \ldots \epsilon_N}} - \langle \widehat{n} \rangle_{\epsilon_1 \ldots \epsilon_N} - 1 = \alpha^2 \left[\frac{\mathcal{F}^{(2)}_{\epsilon_1 \ldots \epsilon_N}\left(\alpha^2\right)}{\mathcal{F}^{(1)}_{\epsilon_1 \ldots \epsilon_N}\left(\alpha^2\right)} - \frac{\mathcal{F}^{(1)}_{\epsilon_1 \ldots \epsilon_N}\left(\alpha^2\right)}{\mathcal{F}_{\epsilon_1 \ldots \epsilon_N}\left(\alpha^2\right)} \right] \ . \tag{11}$$

We find that the photon statistics of these higher-generation Schrödinger-cat states oscillates between super- and sub-Poissonian characters as the intensity of the initial coherent state in the cavity is varied. Also, there are many groups of distinct states with the same photon-number distributions; for example the states $|\Psi_{+-}\rangle$, $|\Psi_{+--}\rangle$ and $|\Psi_{+-+}\rangle$ possess identical $P(n)$. Thus, the elements of these groups of states can only be distinguished by probing properties which depend on the off-diagonal elements of the density matrix in the number basis. Atomic population inversion for two-level atoms interacting with these states, which presents well defined collapses and revivals, depend on $P(n)$ and so does not serve for this purpose. An effective way to distinguish states in such a case is to look at their phase space representations, like the Wigner function.

Finally, we mention that these states have maximum nonclassical depth, $\tau_m = 1$ (the maximum degree of nonclassicality within the phase-space measure of nonclassicality), but they have distinct degrees of nonclassicality within distance type of measures.

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