The role of entanglement in Young interference experiment with two photons[1]

*Leonardo Neves, C. H. Monken and S. Pádua**

Departamento de Física, Universidade Federal de Minas Gerais.
Caixa Postal 702, 30123-970 Belo Horizonte MG, Brazil.
* *spadua@fisica.ufmg.br*

Abstract

In this work, we study the role of quantum entanglement in a double-slit Young interference experiment that uses a pair of photons generated by parametric down-conversion. The quantum state of these photons is calculated for a general case, where they are transmitted through apertures placed in their paths. Considering those apertures as double-slits we show that is possible to treat this system as two *qubits*. In this way we relate the fourth-order interference patterns with the degree of entanglement of twin photons.

1 Introduction

The spontaneous parametric down-conversion (SPDC) is a process in wich a single photon is splitted in a pair of lower-energy photons, due to a interaction with a nonlinear crystal. The twin photons can be entangled in various degrees of freedom, like energy, momentum and polarization and therefore this process is very useful to demonstrate the nonclassical aspects of quantum mechanics.

The quantum state of two photons generated by SPDC considering the monocromatic, paraxial and thin crystal aproximations is given by [1]

$$|\,\Psi\,\rangle = |\,\mathrm{vac}\,\rangle + const. \int d\boldsymbol{q}_1 \int d\boldsymbol{q}_2\, v(\boldsymbol{q}_1+\boldsymbol{q}_2)|\,1\boldsymbol{q}_1\,\rangle|\,1\boldsymbol{q}_2\,\rangle; \tag{1}$$

where $\boldsymbol{q}$ is the transverse component (xy) of the downconverted photons wave vector $\boldsymbol{k}$; $v(\boldsymbol{q})$ is the angular spectrum of the pump beam wich is given by the Fourier transform of his transversal profile, $W(\boldsymbol{\rho})$, at $z=0$ (crystal position); $\boldsymbol{\rho}$ is the transverse component of $\boldsymbol{r}$. Due to the non-factorized dependence of v on $\boldsymbol{q}_1$ and $\boldsymbol{q}_2$, the two-photon state is entangled in momenta space (transversal component). The transference of angular spectrum of the pump beam to the two-photon state permits to control the properties of transversal correlations of these photons by manipulating the pump beam transversal profile.

2 Two-photon state transmitted through apertures

In an experiment where a pair of photons generated by SPDC is transmitted through apertures A_1 and A_2 placed in their paths, we want to know how the two-photon state is affected by those apertures. To obtain it, we compare the expression of the coincidence probability amplitude [2], given by $\langle\,\mathrm{vac}\,|\,\hat{E}_1^{(+)}(\boldsymbol{r}_1)\hat{E}_2^{(+)}(\boldsymbol{r}_2)\,|\,\Psi\,\rangle$, calculated in two symetrical situations showed in Fig. 1 and described below:

1.a) The two-photon state in $z=0$ (crystal position) is substituted in coincidence amplitude above; the eletric field operator propagates from $z=0$ to $z=z_D$ (plane of detectors), being affected by apertures in $z=z_A$. In this case, $|\,\Psi\,\rangle$, given by (1) and the field operator [3] are known.

1.b) The two-photon state is given in $z=z_A$ (plane of apertures); the eletric field operator propagates from $z=z_A$ to $z=z_D$. In this case, the field operator is given by

$$\hat{E}^{(+)}(\boldsymbol{\rho},z) = \int d\boldsymbol{q}\, e^{i[\boldsymbol{q}\cdot\boldsymbol{\rho}-\frac{q^2}{2k}(z-z_A)]}\hat{a}(\boldsymbol{q}) \tag{2}$$

and the two-photon state is unknown. We assume that state must have the following aspect:

$$|\,\Phi\,\rangle = const. \int d\boldsymbol{q}_1 \int d\boldsymbol{q}_2\, \mathcal{F}(\boldsymbol{q}_1,\boldsymbol{q}_2)|\,1\boldsymbol{q}_1\,\rangle|\,1\boldsymbol{q}_2\,\rangle; \tag{3}$$

where $\mathcal{F}(\boldsymbol{q}_1,\boldsymbol{q}_2)$ is the function that characterizes this state and which we want determine.

[1] Leonardo Neves, E. J. S. Fonseca, L. Davidovich, C. H. Monken and S. Pádua, manuscript in preparation (2003).

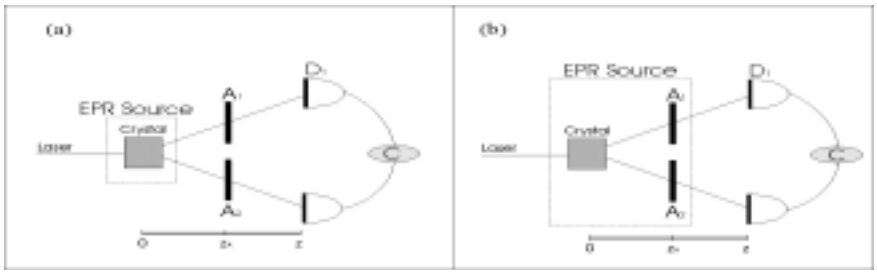


Figure 1: EPR source. In (a) the entangled photons source is the nonlinear crystal. In (b) this source is extended up to apertures.

We can say that in 1.a scheme the crystal is our source of entangled photons and the detection include the apertures. In 1.b scheme this source is extend to plane of apertures. Comparing the coincidence probability amplitude calculated by these two schemes and after a mathematical treatment we obtain

$$\mathcal{F}(\boldsymbol{q}_1,\boldsymbol{q}_2) = const.\int d\boldsymbol{\rho}_1 \int d\boldsymbol{\rho}_2\, A_1(\boldsymbol{\rho}_1)A_2(\boldsymbol{\rho}_2)e^{i\frac{k}{2z_A}\frac{|\boldsymbol{\rho}_2-\boldsymbol{\rho}_1|^2}{\mu_1\mu_2}} W\left(\frac{\boldsymbol{\rho}_1}{\mu_1}+\frac{\boldsymbol{\rho}_2}{\mu_2};z_A\right) e^{-i(\boldsymbol{q}_1\cdot\boldsymbol{\rho}_1+\boldsymbol{q}_2\cdot\boldsymbol{\rho}_2)}; \qquad (4)$$

where $A_i(\boldsymbol{\rho}_i)$ is a function that describe the aperture in the path of photon i. To obtain this expression we have defined new variables $\frac{k}{z_A} \equiv \frac{k_1}{z_{A_1}} + \frac{k_2}{z_{A_2}}$ and $\mu_j \equiv \frac{k}{z_A}\frac{z_{A_j}}{k_j}$ to $(j = 1, 2)$. Thus $\frac{1}{\mu_1}+\frac{1}{\mu_2} = 1$. $W(\boldsymbol{\rho}; z_A)$ is the pump beam transversal profile calculated in $z = z_A$ (plane of the apertures).

The two-photon state (3) with $\mathcal{F}$ given by (4) is a general state for experiments like that in Fig. 1. The control of transversal correlations properties of twin photons will be defined by the form of apertures A_1 and A_2 and the manipulation of the pump beam transversal profile at the plane of those apertures.

3 Interference and entanglement

The Young interference experiment consists to illuminate a double-slit and observe an intensity pattern at a screen on the far side after the slits. This is a standard experiment for studing the transversal correlations properties of light. If the transversal coherence length is greater than the separation between the slits, then we observe an interference pattern. In terms of quantum mechanics, this interference pattern is observed because there is no way of knowing which of the slits the photon passed. If we had the information of which path the photon followed the interference pattern is destroyed.

In this work we consider two double-slits spatially separated and illuminated by twin photons like the scheme showed at Fig. 1. Then we discuss how the quantum state of those photons is related to the results of such experiments.

Let us now calculate the two-photon state after the double-slits (which are identical). To do this we consider the variables in (3) and (4) as one-dimensionals, i.e., $\boldsymbol{\rho} \to x$ e $\boldsymbol{q} \to q$. The transmission function that describes the double-slits is given by:

$$A_i(x_i) = \Pi[(x_i+d)/2a] + \Pi[(x_i-d)/2a]; \qquad \text{with} \quad \Pi[(x_i\pm d)/2a] = \begin{cases} 1, & \mp d - a < x_i < \mp d + a \\ 0, & \text{otherwise;} \end{cases} \qquad (5)$$

where the distance between the slits is $2d$ and its width is $2a$. Then we can write (4) as $\mathcal{F}(q_1, q_2) = const.\sum_{\varepsilon,\varepsilon'=\pm} F_{\varepsilon\varepsilon'}(q_1, q_2)$, where

$$F_{\pm\pm}(q_1,q_2) \approx 4a^2 W(\pm d; z_A)e^{\mp iq_1 d}\,\mathrm{sinc}\,(q_1 a)e^{\mp iq_2 d}\,\mathrm{sinc}\,(q_2 a); \qquad (6)$$

$$F_{\pm\mp}(q_1,q_2) \approx 4a^2 W(0; z_A)e^{\mp iq_1 d}\,\mathrm{sinc}\,(q_1 a)e^{\pm iq_2 d}\,\mathrm{sinc}\,(q_2 a), \qquad (7)$$

where $\mathrm{sinc}\, x = \sin x/x$. Now we define the normalized states (up to an arbitrary phase):

$$|\pm\rangle_j \equiv \left(\frac{a}{\pi}\right)^{\frac{1}{2}} \int dq_j\, e^{\mp iq_j d}\,\mathrm{sinc}\,(q_j a)|\, 1q_j\,\rangle \qquad j = 1, 2; \qquad (8)$$

These states are orthogonals (${}_j\langle + | - \rangle_j = 0$) and have a simple meaning: $| + \rangle_j$ ($| - \rangle_j$) represents the photon j passing through up-slit (down-slit).

So the two-photon state (3) after the double-slits will be written like a discrete sum

$$| \Phi \rangle = N \sum_{\varepsilon=\pm} \sum_{\varepsilon'=\pm} C_{\varepsilon\varepsilon'} | \varepsilon \rangle_1 \otimes | \varepsilon' \rangle_2; \tag{9}$$

where N is a normalization constant and the probability amplitudes $C_{\varepsilon\varepsilon'}$ are given by

$$C_{\pm\pm} = W(\pm d; z_A); \qquad C_{+-} = C_{-+} = W(0; z_A). \tag{10}$$

These coeficients are the values of the pump beam transversal profile in the plane of the double-slits ($z = z_A$), measured at the "slits positions" ($x = \pm d$) or at the "dark region" between the slits ($x = 0$). Thus, the two-photon system can be treated like a two qubits system and by manipulating the pump beam transversal profile, we can generate entangled states of two qubits.

Writing the two-photon state in (9) in terms of the Schmidt decomposition [4], we have

$$| \Phi \rangle = \sqrt{\lambda_+} | \eta_+ \rangle_1 \otimes | \chi_+ \rangle_2 + \sqrt{\lambda_-} | \eta_- \rangle_1 \otimes | \chi_- \rangle_2, \tag{11}$$

where $| \eta_\pm \rangle_1 = \frac{1}{\sqrt{2}}(| + \rangle_1 \pm | - \rangle_1)$ and $| \chi_\pm \rangle_2 = \frac{1}{\sqrt{2}}(| + \rangle_2 \pm | - \rangle_2)$. The Schmidt coeficients are given by

$$\sqrt{\lambda_+} = \frac{W(d, z_A) + W(0, z_A)}{\sqrt{2([W(d, z_A)]^2 + [W(0, z_A)]^2)}}\,, \qquad \sqrt{\lambda_-} = \frac{W(d, z_A) - W(0, z_A)}{\sqrt{2([W(d, z_A)]^2 + [W(0, z_A)]^2)}}\,. \tag{12}$$

A good entanglement measurement for pure states of two qubits is the *concurrence* [5], which in terms of Schmidt coeficients is given by $\mathcal{C}(\Phi) = 2\left|\sqrt{\lambda_-\lambda_+}\right|$. So the entanglement degree of the twin photons will be

$$\mathcal{C}(\Phi) = \left| \frac{[W(d, z_A)]^2 - [W(0, z_A)]^2}{[W(d, z_A)]^2 + [W(0, z_A)]^2} \right|. \tag{13}$$

Now we see explicitly that the manipulation of pump beam transversal profile at the plane of the apertures, controls the entanglement degree of the twin photons.

Three types of two-photon states can be prepared:

Product state ($\mathcal{C}(\Phi) = 0$)

To prepare a product state we must have $W(\pm d, z_A) = W(0, z_A)$. So, with a wide gaussian pump beam transversal profile (constant at $x = \pm d$ and $x = 0$), we generate a two-photon state $| \Phi \rangle = | \eta_+ \rangle_1 \otimes | \chi_+ \rangle_2$. In this state each photon of the pair must interfere independently. The fourth-order interference pattern will have periodicity proportinal to λ, the wavelength of each photon component of twin photons.

Partially entangled states ($0 < \mathcal{C}(\Phi) < 1$)

With a pump beam transversal profile (at $z = z_A$) such that $W(\pm d, z_A) \neq W(0, z_A)$, we can prepare a partially entangled state. The fourth order interfence pattern must have an indefinite periodicity because this state has characteristics of product states and maximally entangled states.

Maximally entangled states ($\mathcal{C}(\Phi) = 1$)

In (13) we see that there are two distinct ways to prepare a maximally entangled states:

a) Focus the pump beam transversal profile at $x = a$ such that $W(\pm d, z_A) = 0$, and

$$| \Phi_F \rangle = \frac{1}{\sqrt{2}}(| + \rangle_1 \otimes | - \rangle_2 + | - \rangle_1 \otimes | + \rangle_2).$$

b) Project the image of a wire at the slits' plane such that, and $W(0, z_A) = 0$, and

$$| \Phi_W \rangle = \frac{1}{\sqrt{2}}(| + \rangle_1 \otimes | + \rangle_2 + | - \rangle_1 \otimes | - \rangle_2).$$

These states are analogous of the Bell's states. Fig. 2 ilustrates the type of correlations between the twin photons and the respective pump beam transversal profile that generate them. In these states, the photon pair interfere like one particle because there is no way to know which pair of slits the photon pair has passed. Thus, the fourth order interference pattern will have a periodicity proportional to $\lambda/2$ (nonlocal de Broglie wavelength [6]). This demonstrates a nonlocal behaviour.

In the simulation of the experiment (made by E. Fonseca [7] through theoretical expression of coincidence probability) showed at Fig. 3, we see the fourth order interference patterns for various two-photon states. The graphics confirm what we affirmed above.

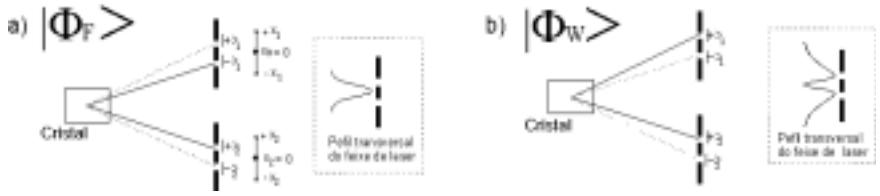


Figure 2: Transverse profiles of the pump beam (in the plane of the double-slits) that maximize the entanglement of two-photon state.

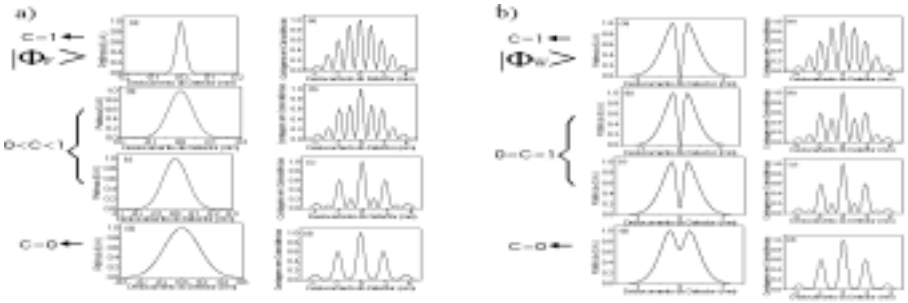


Figure 3: Relation between entanglement degree of two-photon states and fourth order interference patterns.

4 Conclusion

We have shown a general expression for a two-photon state generated by SPDC and transmitted through the apertures. The state's calculation of the twin photons transmitted through the double-slits shows that is possible treat them like a two qubits system. In this way, we calculate the entanglement degree by concurrence and show that it is controlled by manipulation of pump beam transversal profile. A qualitative analysis has been given relating the fourth-order interference patterns with the entanglement degree of the twin photons.

Acknowledgements

This work was partially supported by Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq), Coordenação de Aperfeiçoamento de Pessoal de Nível Superior (CAPES), Brazil.

References

[1] C. H. Monken, P. H. S. Ribeiro and S. Pádua, *Phys. Rev. A* **57**, 3123 (1998).

[2] M. O. Scully and M. S. Zubairy, *Quantum Optics*, (Cambridge University Press, Cambridge, 1997).

[3] C. H. Monken, course notes - Physics Department - UFMG, unpublished (1999).

[4] A. Ekert and P. L. Knight, *Am. J. Phys.* **63**, 415 (1994).

[5] S. Hill and W. K. Wooters, *Phys. Rev. Lett* **78**, 5022 (1997).

[6] E. J. S. Fonseca, Z. Paulini, P. Nussenzweig, C. H. Monken and S. Pádua, *Phys. Rev. A* **63**, 043819 (2001).

[7] E. J. S. Fonseca, Doctor Thesis - Physics Department - UFMG, unpublished (2000).